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一类非线性系统的自适应控制及其在发动机中的应用*

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摘 要: 讨论了一类不确定性严格反馈非线性系统的鲁棒控制问题。结合 H_∞控制和自适应控制, 提出基于一种反演 (Backstepping) 技术的鲁棒控制器。将广义不确定项参数化, 提出了状态反馈控制器的算法, 反演迭代设计不仅避免了求解 HJI不等式设计控制器的困难, 而且保证了闭环系统取得 H_∞的性能指标。基于航空发动机模型的仿真结果证明了理论推导的有效性。

关键词: 航空发动机; 非线性; 反演⁺; 自适应控制

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An adaptive control for a class of nonlinear systems and application to aeroengine control

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Abstract A discussion is devoted to an adaptive robust control method for a class of strict feedback uncertain nonlinear system. Combined adaptive control and H_∞ theory, a robust controller based on backstepping technique is proposed. State feedback algorithm is discussed with general uncertainties parameterized. Recursive design based on backstepping technique can avoid solving the HJI inequalities and guarantee closed loop system to achieve H_∞ performers.

Key words Aeroengine; Nonlinearity; Backsteppping⁺; Adaptive control

1 引 言

航空发动机是非常复杂的非线性对象, 其参数变化范围很大, 精确的数学模型很难建立, 不论采用什么控制方法, 都必须确保所设计的发动机控制系统具有很强的鲁棒稳定性。

反演设计方法是一种很有效的非线性设计方法, 目前为止已经得到了广泛应用, 如机器人控制、电机控制、飞行控制等。

本文讨论了一类不确定性严格反馈非线性系统的鲁棒控制问题。参考文献 [2]的研究成果结合 H_∞控制和自适应控制, 提出基于一种反演技术的鲁棒控制器。将广义不确定项参数化, 提出了状态反馈控制器的算法, 反演迭代设计不仅避免了求解 HJI不等式设计控制器的困难, 而且保证了闭环系统取得 H_∞的性能指标。将此方法应用到航空发动机供油量控制中, 仿真结果证明了理论推导的有效性。

2 问题描述

考虑不确定 SISO严格反馈型非线性系统描述如下

$$
\begin{aligned}
\dot{x}_i &= f_i(\bar{x}_i, w) + g_i(\bar{x}_i, w)x_{i+1} + d_i(t) \\
\dot{x}_n &= f_n(x, w) + g_n(x, w)u + d_n(t) \\
y &= x_1
\end{aligned}
\tag{1}
$$

式中 $x = [x_1, \cdots, x_n]^{\mathrm{T}} \in R^n$ 为系统状态, $u \in R$ 是标量控制输入, $y \in R$ 是系统输出; $w \in W \subset R^q$ 是 q 维参数

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不确定向量, W 是一个紧集; $d_i(t)$, $i=1,2,\cdots,n$ 为系统时变干扰不确定项。

定义 $\bar{x}_i=[x_1,\cdots,x_i]^{\mathrm{T}}$ $f_i(\bar{x}_i,w)$ 为未知非线性光滑函数, $g_i(\bar{x}_i,w)$ 为未知光滑虚拟控制增益函数。对此系统做如下假设:

假设 2.1

f_i、g_i 未知非线性光滑函数可以描述为以下形式

$$f_i(\bar{x}_i,w)=f_{in}(\bar{x}_i)+\Delta f_i(\bar{x}_i,w)$$
$$g_i(\bar{x}_i,w)=g_{in}(\bar{x}_i)+\Delta g_i(\bar{x}_i,w),\ i=1,2,\cdots,n \tag{2}$$

式中 $f_{in}(\bar{x}_i)$, $g_{in}(\bar{x}_i)$ 为标称函数, $f_i(0,w)=0$。

$\Delta f_i(\bar{x}_i,w)$, $\Delta g_i(\bar{x}_i,w)$ 满足

$$\begin{cases}\Delta f_i(\bar{x}_i,w)=\theta^{\mathrm{T}}\Phi_i(\bar{x}_i),\\ g_i^L(\bar{x}_i,w)\leqslant\Delta g_i(\bar{x}_i,w)\leqslant g_i^U(\bar{x}_i,w)\end{cases} \tag{3}$$

式中 $\theta=[\theta_1,\cdots,\theta_m]^{\mathrm{T}}\in R^m$, 且 $\theta>0$, $\Phi_i(\bar{x}_i)$ 为 m 维光滑函数向量。

假设 2.2

不确定项 $\Delta_i(\bar{x}_i,w)=\Delta f_i(\bar{x}_i,w)+\Delta g_i(\bar{x}_i,w)x_{i+1}$, $i=1,2,\cdots,n-1$, 且有

$$\mu_i=\Delta g_i(\bar{x}_i,w)x_{i+1},\ |\mu_i|\leqslant\varepsilon_i,\ \varepsilon_i>0 \tag{4}$$

假设 2.3

在系统 (1)中, 扰动不确定项 $d_i(t)\in L_2[0,T]$, $i=1,2,\cdots,n$。

为简明起见, 在不引起混淆的情况下, 我们将省略函数表示中的变量。

3 控制器设计及稳定性分析

第 1步, 引入误差变量

$$z_1=x_1,\ z_2=x_2-\alpha_1$$

记 $\bar{\Delta}_1=\Delta_1$, $\phi_1=\Phi_1$, 其中

$$\alpha_1=g_{1n}^{-1}\left[-k_1z_1-f_{1n}-\hat{\theta}^{\mathrm{T}}\phi_1-\frac{1}{2\gamma^2}z_1\right] \tag{5}$$

α_1 是虚拟控制, $k_1>0$ 为设计参数, $\gamma>0$ 是任意给定的参数。对 z_1 求时间的导数

$$\dot{z}_1=g_{1n}(z_2+\alpha_1)+f_{1n}+\Delta_1+d_1(t) \tag{6}$$

考虑如下 Lyapunov函数

$$V_1=\frac{1}{2}z_1^2+\frac{1}{2}\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\tilde{\theta} \tag{7}$$

式中设计参数 Γ 为 m 维正定矩阵, $\hat{\theta}$ 是对 θ 的估计, 记 $\tilde{\theta}=\hat{\theta}-\theta$。

对 V_1 求时间导数, 可得

$$\begin{aligned}\dot{V}_1&=z_1\dot{z}_1+\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}}=z_1[g_{1n}(z_2+\alpha_1)+\\&f_{1n}+\Delta_1+d_1]+\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}}=\\&z_1\left[g_{1n}z_2-k_1z_1-\frac{z_1}{2\gamma^2}\right]+z_1(\Delta_1+d_1-\hat{\theta}^{\mathrm{T}}\phi_1)+\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}}\leqslant\\&-k_1z_1^2+z_1g_{1n}z_2+|z_1|v_1-\frac{z_1^2}{2\gamma^2}+\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}\dot{\hat{\theta}}-\tilde{\theta}^{\mathrm{T}}\phi_1z_1\end{aligned} \tag{8}$$

式中 $v_1=|\varepsilon'_1|+|\bar{d}_1|$, $\varepsilon_1=\varepsilon'_1$, $\bar{d}_1=d_1$。

参考文献 [2]的方法, 运用 Schwarz不等式, 可得

$$v_1|z_1|=(|\varepsilon'_1|+|\bar{d}_1|)|z_1|\leqslant$$
$$\frac{1}{2\gamma^2}z_1^2+\gamma^2(\varepsilon_1'^2+\bar{d}_1^2)\leqslant\frac{1}{2\gamma^2}z_1^2+\delta_1 \tag{9}$$

式中 $\delta_1=\gamma^2(\varepsilon_1^2+d_1^2)$。

记 $\tau_1=\Gamma\phi_1z_1$, 将式 (9)代入式 (8), 有

$$\dot{V}_1\leqslant-k_1z_1^2+z_1g_{1n}z_2+\tilde{\theta}^{\mathrm{T}}\Gamma^{-1}(\dot{\hat{\theta}}-\tau_1)+\delta_1 \tag{10}$$

第 2步, 对 α_1 求时间导数可得

$$\begin{aligned}\dot{\alpha}_1&=\frac{\partial\alpha_1}{\partial x_1}\dot{x}_1+\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}=\\&\frac{\partial\alpha_1}{\partial x_1}(f_{1n}+g_{1n}x_2+\Delta_1)+\frac{\partial\alpha_1}{\partial x_1}d_1+\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}\end{aligned} \tag{11}$$

定义误差变量 $z_3=x_3-\alpha_2$, 并对 z_2 求导可得

$$\begin{aligned}\dot{z}_2&=g_{2n}(\bar{x}_2)x_3+f_{2n}(\bar{x}_2)+\Delta_2+d_2-\dot{\alpha}_1=\\&g_{2n}(z_3+\alpha_2)+\frac{\partial\alpha_2}{\partial\hat{\theta}}\tau_2-\frac{1}{2\gamma^2}\left[\frac{\partial\alpha_1}{\partial x_1}\right]^2z_2+\\&f_{2n}(\bar{x}_2)-\frac{\partial\alpha_1}{\partial x_1}(f_{1n}(x_1)+g_{1n}(x_1)x_2)-\\&\frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2+\frac{1}{2\gamma^2}\left[\frac{\partial\alpha_1}{\partial x_1}\right]^2z_2-\frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}}+\\&\Delta_2-\frac{\partial\alpha_1}{\partial x_1}\Delta_1+d_2-\frac{\partial\alpha_1}{\partial x_1}d_1\end{aligned} \tag{12}$$

$$\bar{\dot{\alpha}}=\left[\frac{\partial\alpha_1}{\partial x_1}\right](g_{1n}x_2+f_{1n})+\left[\frac{\partial\alpha_1}{\partial\hat{\theta}}\right]\tau_2-\frac{1}{2\gamma^2}\left[\frac{\partial\alpha_1}{\partial x_1}\right]^2z_2$$

$\bar{\Delta}_2=\Delta_2-\dfrac{\partial\alpha_1}{\partial x_1}\Delta_1$, $\bar{d}_2=d_2-\dfrac{\partial\alpha_1}{\partial x_1}d_1$。

由假设 2.2, 可得

$$\Delta f_2-\frac{\partial\alpha_1}{\partial x_1}\Delta f_1=\theta^{\mathrm{T}}\left[\Phi_2-\frac{\partial\alpha_1}{\partial x_1}\Phi_1\right]=\theta^{\mathrm{T}}\phi_1,$$
$$\left|\mu_2-\frac{\partial\alpha_1}{\partial x_1}\mu_1\right|\leqslant\varepsilon_2+\left|\frac{\partial\alpha_1}{\partial x_1}\right|\varepsilon_1=\varepsilon'_2 \tag{13}$$

式中 $\phi_2=\Phi_2-\dfrac{\partial\alpha_1}{\partial x_1}\Phi_1$, $\varepsilon'_2=\varepsilon_2+\left|\dfrac{\partial\alpha_1}{\partial x_1}\right|\varepsilon_1$。

式(12)可写成

$$\begin{aligned}\dot z_2 = {} & g_{2n}(z_3+\alpha_2)+\frac{\partial\alpha_1}{\partial\hat\theta}(\tau_2-\dot{\hat\theta}) - \\ & \frac{1}{2\gamma^2}\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2 + f_{2n}+\bar\Delta_2+\bar d_2-\dot{\bar\alpha}_1 \end{aligned}\qquad(14)$$

考虑如下 Lyapunov函数

$$V_2 = V_1 + \frac{1}{2}z_2^2 \qquad(15)$$

对 V_2 求时间导数,可得

$$\begin{aligned}\dot V_2 \leqslant {} & -k_1z_1^2 + z_1 g_{1n} z_2 + \theta^T\Gamma^{-1}(\dot{\hat\theta}-\tau_1)+\delta_1 - \\ & z_2\frac{\partial\alpha_1}{\partial\hat\theta}(\dot{\hat\theta}-\tau_2) - \frac{1}{2\gamma^2}\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 + \\ & z_2[g_2(z_3+\alpha_2)+f_{2n}+\bar\Delta_2+\bar d_2-\dot{\bar\alpha}_1] \end{aligned}\qquad(16)$$

选择虚拟控制 α_2

$$\alpha_2 = g_{2n}^{-1}\left[-k_2z_2 - f_{2n} - z_1 g_{1n} - \hat\theta^T\phi_2 - \frac{1}{2\gamma^2}z_2 + \dot{\bar\alpha}_1\right] \qquad(17)$$

将式(17)代入式(16)可得

$$\begin{aligned}\dot V_2 \leqslant {} & -\sum_{j=1}^{2} k_j z_j^2 + z_2 g_{2n} z_3 + \theta^T\Gamma^{-1}(\dot{\hat\theta}-\tau_1) + \\ & \delta_1 - z_2\frac{\partial\alpha_1}{\partial\hat\theta}(\dot{\hat\theta}-\tau_2) - \frac{1}{2\gamma^2}\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 - \\ & \frac{1}{2\gamma^2}z_2^2 + |z_2|(|\varepsilon'_2|+|\bar d_2|) - \theta^T\phi_2 z_2 \end{aligned}\qquad(18)$$

记 $v_2 = |\varepsilon'_2|+|\bar d_2|$,由于

$$v_2|z_2| \leqslant \frac{1}{2\gamma^2}z_2^2 + \frac{1}{2\gamma^2}\left(\frac{\partial\alpha_1}{\partial x_1}\right)^2 z_2^2 + \gamma^2\left[\sum_{j=1}^{2}(\varepsilon_j^2+d_j^2)\right] \qquad(19)$$

将式(19)代入式(18)可得

$$\begin{aligned}\dot V_2 \leqslant {} & -\sum_{j=1}^{2} k_j z_j^2 + z_2 g_{2n} z_3 + \theta^T\Gamma^{-1}(\dot{\hat\theta}-\tau_1-\Gamma\phi_2 z_2) + \\ & \delta_1 - z_2 \frac{\partial\alpha_1}{\partial\hat\theta}(\dot{\hat\theta}-\tau_2)+\gamma^2\left[\sum_{j=1}^{2}(\varepsilon_j^2+d_j^2)\right] \leqslant \\ & -\sum_{j=1}^{2} k_j z_j^2 + g_{2n}z_2z_3 + \left[\Gamma^{-1}\theta - z_2\frac{\partial\alpha_1}{\partial\hat\theta}\right](\dot{\hat\theta}-\tau_2) + \delta_2 \end{aligned}\qquad(20)$$

式中 $\tau_2 = \tau_1 + \Gamma\phi_2 z_2$

$$\delta_2 = \delta_1 + \gamma^2\left[\sum_{j=1}^{2}(\varepsilon_j^2+d_j^2)\right]。$$

第 $i(3\leqslant i\leqslant n-1)$步,记 $\bar\Delta_i = \Delta_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j$, $\bar d_i = d_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}d_j$, 由假设 2.2有

$$\begin{aligned} & \Delta f_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta f_j = \theta^T\left[\varphi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\varphi_j\right] = \theta^T\phi_i, \\ & \left|\mu_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\mu_j\right| = \varepsilon_i + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varepsilon_j = \varepsilon'_i \end{aligned}\qquad(21)$$

式中 $\phi_i = \varphi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_i}\varphi_j$

$$\varepsilon'_i = \varepsilon_i + \sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\varepsilon_j$$

定义误差变量 $z_i = x_i - \alpha_{i-1}$, $z_{i+1}=x_{i+1}-\alpha_i$, 选择虚拟控制函数

$$a_i = g_{in}^{-1}\left[-k_iz_i - f_{in} - \hat\theta^T\phi_i - g_{(i-1)n}z_{i-1} - \frac{z_i}{2\gamma^2}+\dot{\bar\alpha}_{i-1}\right] \qquad(22)$$

式中
$$\begin{aligned}\dot{\bar\alpha}_{i-1} = {} & \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(f_{jn}+g_{jn}x_{j+1}) + \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\tau_i + \\ & \sum_{j=1}^{i-2} z_{j+1}\frac{\partial\alpha_j}{\partial\hat\theta}\Gamma\phi_i - \frac{1}{2\gamma^2}\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2 z_i \end{aligned}$$

求 α_{i-1}的时间导数

$$\begin{aligned}\dot\alpha_{i-1} = {} & \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\dot x_j + \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(g_{jn}x_{j+1}+f_{in}) + \\ & \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}d_j + \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta} \end{aligned}\qquad(23)$$

对 z_i 求时间的导数可得

$$\begin{aligned}\dot z_i = {} & f_{in}+g_{in}x_{i+1}+\Delta_i + d_i - \dot\alpha_{i-1} = \\ & f_{in}+g_{in}(z_{i+1}+\alpha_i)+\bar\Delta_i+\bar d_i + \frac{\partial\alpha_{i-1}}{\partial\hat\theta}(\tau_i-\dot{\hat\theta}) - \\ & \frac{1}{2\gamma^2}\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2 z_i + \sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat\theta}\Gamma\phi_i - \dot{\bar\alpha}_{i-1} \end{aligned}\qquad(24)$$

考虑如下 Lyapunov函数

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \qquad(25)$$

对 V_i 求时间导数,并将式(23)和式(24)代入得

$$\begin{aligned}\dot V_i = {} & \dot V_{i-1} + z_i\dot z_i \leqslant -\sum_{j=1}^{i-1} k_j z_j^2 + g_{(i-1)n}z_{i-1}z_i + \\ & \theta^T\Gamma^{-1}(\dot{\hat\theta}-\tau_{i-1}) - \frac{1}{2\gamma^2}\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2 z_i^2 + \\ & z_i\sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat\theta}\Gamma\phi_i + \sum_{j=1}^{i-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat\theta}(\tau_{i-1}-\dot{\hat\theta}) + \\ & z_i\frac{\partial\alpha_{i-1}}{\partial\hat\theta}(\tau_i-\dot{\hat\theta}) + z_i[(z_{i+1}+\alpha_i)g_{in} + \\ & f_{in}+\bar\Delta_i+\bar d_i - \dot{\bar\alpha}_{i-1}] + \delta_{i-1} \leqslant \end{aligned}$$

$$-\sum_{j=1}^{i}k_j z_j^2 + g_{in}z_i z_{i+1} + \theta^{T}\Gamma^{-1}(\dot{\hat{\theta}} - \tau_{i-1}) -$$

$$\sum_{j=1}^{i-2} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_{i-1} - \Gamma\phi_i z_i) -$$

$$z_i\frac{\partial \alpha_{i-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}}-\tau_i) - \frac{1}{2\gamma^2}\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|^2 z_i^2 +$$

$$\delta_{i-1} - \frac{1}{2\gamma^2}z_i^2 + |z_i|(|\varepsilon'_i| + |\bar{d}_i|) - \theta^{T}\phi_i z_i$$

记 $\delta_i = \delta_{i-1} + \gamma^2\left(\sum_{j=1}^{i}(\varepsilon_j^2 + d_j^2)\right)$, $v_i = |\varepsilon'_i| + |\bar{d}_i|$, $\tau_i = \tau_{i-1} + \Gamma\phi_i z_i$。

运用 Schwarz不等式

$$v_i|z_i| \leqslant \frac{1}{2\gamma^2}z_i^2 + \frac{1}{2\gamma^2}\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|^2 z_i^2 + \gamma^2\left(\sum_{j=1}^{i}(\varepsilon_j^2+d_j^2)\right) \tag{27}$$

将式(27)代入式(26), 可得

$$\dot{V}_i \leqslant -\sum_{j=1}^i k_j z_j^2 + g_{in}z_i z_{i+1} + \theta^{T}\Gamma^{-1}(\dot{\hat{\theta}}-\tau_{i-1} - \Gamma\phi_i z_i) - \sum_{j=1}^{i-2} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}(\dot{\hat{\theta}}-\tau_{i-1}-\Gamma\phi_i z_i) - z_i\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}}-\tau_i)+\delta_i \leqslant$$

$$-\sum_{j=1}^i k_j z_j^2 + g_{in}z_iz_{i+1} + \left(\theta^{T}\Gamma^{-1} - \sum_{j=1}^{i-1} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\right)(\dot{\hat{\theta}}-\tau_i)+\delta_i \tag{28}$$

第 n步, 定义误差变量 $z_n = x_n - \alpha_{n-1}$, 并对 z_n 求导得

$$\dot{z}_n = f_{nn}(\bar{x}_n) + \Delta f_n + [g_{nn}(\bar{x}_n) + \Delta g_n(\bar{x}_n, t)]u + d_n - \dot{\alpha}_{n-1} \tag{29}$$

记 $\bar{\Delta}_n = \Delta_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\Delta_j$, $\bar{d}_n = d_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}d_j$

且有 $\varepsilon_n = 0$, 选择控制输入 u

$$u = u_c + u_r \tag{30}$$

$$u_c = \alpha_n = g_{nn}^{-1}\left(-k_n z_n - f_{nn}(x) - g_{(n-1)n}z_{n-1} - \hat{\theta}^{T}\phi_n - \frac{1}{2\gamma^2}z_n + \bar{\dot{\alpha}}_{n-1}\right) \tag{31}$$

$$u_r = \tau v = [g_{nn} + \xi]^{-1}v,\quad \xi = g_n^{L},$$

$$v = \begin{cases} -g_n^{U} u_c & z_n u_c > 0 \\ -g_n^{L} u_c & z_n u_c < 0\end{cases} \tag{32}$$

式中 k_n 为设计参数, $\phi_n = \Phi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\Phi_j$

$$\bar{\dot{\alpha}}_{n-1} = \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(f_{jn} + g_{jn}x_{j+1}) + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\tau_n + \sum_{j=1}^{n-2}z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}\Gamma\phi_n - \frac{1}{2\gamma^2}\sum_{j=1}^{n-1}\left|\frac{\partial\alpha_{n-1}}{\partial x_j}\right|^2 z_n \tag{33}$$

选择自适应律

$$\dot{\hat{\theta}} = \tau_n = \tau_{n-1} + \Gamma\phi_n z_n \tag{34}$$

考虑如下 Lyapunov函数

$$V_n = V_{n-1} + \frac{1}{2} z_n^2 \tag{35}$$

对 V_n 取时间导数, 并将式(30)~(34)代入

$$\dot{V}_n \leqslant -\sum_{j=1}^n k_j z_j^2 + \Gamma^{-1}\theta(\dot{\hat{\theta}} - \tau_{n-1} - \Gamma\phi_n z_n) + \sum_{j=1}^{n-2} z_{j+1}\frac{\partial\alpha_j}{\partial\hat{\theta}}(\dot{\hat{\theta}}-\tau_{n-1}-\Gamma\phi_n z_n) + z_n\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}(\tau_n - \dot{\hat{\theta}}) + z_n\Delta g_n u_c + z_n v(g_{nn}+\Delta g_n)\tau + \delta_n \tag{36}$$

$$\delta_n = \delta_{n-1} + \gamma^2\left(\sum_{j=1}^{n-1}(\varepsilon_j^2 + d_j^2)\right)$$

由式(32), 显然有

$z_n\Delta g_n u_c + z_n v(g_{nn}+\Delta g_n)\tau \leqslant 0$, 可得

$$\dot{V}_n \leqslant -\sum_{j=1}^n k_j z_j^2 + \gamma^2\sum_{j=1}^n((n+1-j)(\varepsilon_j^2+d_j^2)) \leqslant -\sum_{j=1}^n k_j z_j^2 + n\gamma^2\|\omega\|^2 \tag{37}$$

$\omega = [\varepsilon_1, \dots, \varepsilon_n, d_1, \dots, d_n]^{T}$。记 $Q = \mathrm{diag}(k_1, k_2, \dots, k_n)$, 将式(37)两边积分, 可得

$$\int_0^T\|z\|^2 \mathrm{d}t \leqslant n\lambda_{\min}^{-1}(Q)\gamma^2\int_0^T\|\omega\|^2\mathrm{d}t + \lambda_{\min}^{-1}(Q)V(0) \tag{38}$$

定理 3.1在假设 2.1~2.3成立的前提下, 选取鲁棒控制器(30)~(33)和参数自适应律(34)可以保证闭环系统一致最终有界。在给定 $\gamma > 0$, 实现对外部干扰 d_i 和不确定项的抑制, 系统可达到 H_∞ 的性能指标。

4 发动机控制及仿真

某型发动机状态传递函数如下[3]

$$\frac{y_m}{r} = \frac{4(s+2.5)}{s^2+7.0s+10.0} \tag{39}$$

其中, 输入为供油量, 输出为低压转子转速。考虑外界干扰和系统非线性不确定性, 表示成状态空间的形式为

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(x_1, x_2) + g(x_1, x_2)u + d \qquad (40)$$

$f(x_1, x_2) = f_n(x_1, x_2) + \Delta f(x_1, x_2)$, 标称值 $f(x_1, x_2) = -10.0x_1 - 7.0x_2$, 对象不确定性 $\Delta f(x_1, x_2) = 2.0x_1 \sin t + 1.0x_2 \cos t$。

控制增益 $g(x_1, x_2) = 1$, 不确定性 $\Delta g = 0.1\sin t$, $d = (0.1x_1 + 0.05x_2)\cos t$。

外界扰动对象不确定性参数化为 $\Delta f(x_1, x_2) = \theta_1 x_1 + \theta_2 x_2$, 且有 $-0.1 \leqslant \Delta g \leqslant 0.1$。

通过仿真, 控制参数选择为 $k_1 = 2$, $k_2 = 4$, $\Gamma = \mathrm{diag}(4, 1)$, $\gamma = 0.5$。初始条件为 $x_1(0) = 1.0$, $x_2(0) = 0$。

图 1为闭环系统的状态曲线及系统输出, 图 2为控制输入。仿真结果表明此方法, 发动机系统存在不确定性和外界扰动的情况下, 闭环系统可以取得 H_∞ 性能指标。

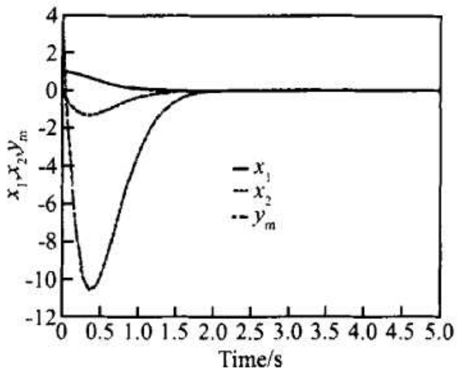

Fig 1 States of closed loop system and output

5 结 论

本文讨论了一类不确定性严格反馈非线性系统的鲁棒控制问题。结合 H_∞ 控制和自适应控制, 提出基于一种反演技术的鲁棒控制器。将广义不确定项参数化, 提出了状态反馈控制器的算法, 反演迭代设计不仅避免了求解 HJI不等式设计控制器的困难, 而且保证了闭环系统取得 H_∞ 的性能指标。

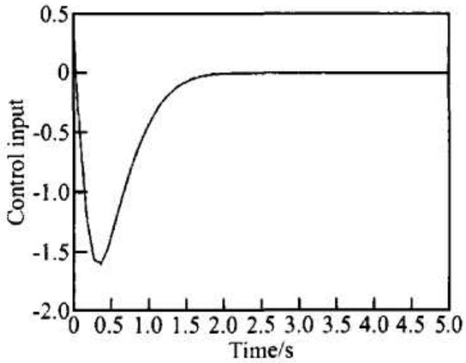

Fig 2 Control input

将此方法应用到航空发动机供油量控制中, 仿真结果证明了此方法具有良好的动态品质和较强的鲁棒性。

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