变容腔燃烧室的不稳定性分析

郜 冶

摘 要

随着装药的燃烧，固体火箭发动机燃烧室容腔不断地扩大，质量补充的注入速度也受到装药燃速的影响。本文利用 Van Moorhem 最近提出的平均处理法，从理论上定性分析了一维管流通道随时间变化和燃烧边界的运动对燃烧室不稳定性的作用。

符 号 表

σ	控制体表面
$\vec{n}$	控制面外法向单位向量
v	气体运动速度
u	控制面运动速度
R	气体常数，装药燃速
m	单位控制体表面实际加入质量
ε	正比于扰动量的小参数 $\lim\limits_{\substack{\varepsilon\to 0\\ M\to 0}}\frac{\varepsilon}{M}=0$
m_R	由(2—14)式定义
L	內通道长度
s	管形药周边长
A	通道横截面积
C_p	定压比热
C_v	定容比热
γ	比热比
ρ	密度
T	温度
e	滞止內能
M	马赫数
a	音速
h	滞止焓
P	压力
t	时间
K	复波数
ⓗ	由(2—25)式定义
I_i	第i项积分
$i=\sqrt{-1}$	
α	声能增长率
x	笛卡尔坐标
Q	任一标量变量
<>	平均值
$E_e^2=\int_0^L \hat{P}_e^2 dx_1$	

上标：

^	复振幅
(1)	一阶扰动量
(0)	稳态量
(0、0)、(1、0)	沒有气体流动时的値
(0、1)、(1、1)	有气体流动时的値

下标：

e	e阶振模值
i	坐标张量分量
j	坐标张量分量
s	燃烧表面值

一、引 言

Culick 利用边界上有质量加入的一维平均流守恒方程，考虑了气体流动与燃烧之间的声

耦合产生的声能增益，燃烧室已不再简单地看作一般的声腔。由于引用了平均流概念，可认为燃烧区边界上的诸因素对流场各空间点的影响是均匀的，在这样简化的边界条件下使得复波数的解成为可能。但是，装药以一定的速率燃烧，管截面不断随时间增大，所取控制体边界也在不断地运动，描述波与燃烧之间相互作用的非齐次方程将变得更为复杂。Van Moorhem 提出的平均处理法，为我们研究容腔变化和边界运动对燃烧室稳定性的影响提供了方法。

二、声波方程和边界条件

图(1)所示控制面σ所包围的燃气。控制面是可变形的又是可穿透的。某瞬时控制面上任意点的速度矢量用$\vec{u}$表示，该瞬时流经这个位置的燃气速度矢量用$\vec{v}$表示，$\vec{v}\cdot\vec{n}$和$\vec{u}\cdot\vec{n}$表示$\vec{v}$和$\vec{u}$在表面微元$\Delta\sigma$外法向的投影。当$\vec{v}\cdot\vec{n}=\vec{u}\cdot\vec{n}$时，$\sigma$是不可穿透的，即没有质量穿过控制面。气体动力学三维守恒方程为

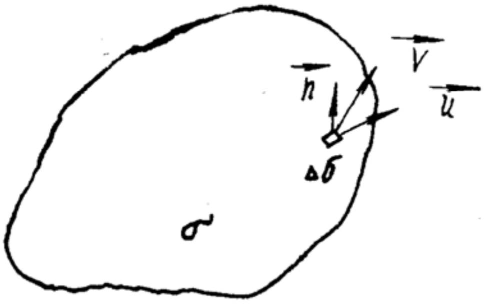

图 1

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x_j}(\rho v_j)=0 \tag{1}$$

$$\frac{\partial(\rho v_i)}{\partial t}+\frac{\partial}{\partial x_j}(\rho v_i v_j)+\frac{\partial p}{\partial x_i}=0 \tag{2}$$

$$\frac{\partial(\rho e)}{\partial t}+\frac{\partial}{\partial x_j}(\rho v_j e)+\frac{\partial}{\partial x_j}(p v_j)=0 \tag{3}$$

在边界上

$$\rho(v_i-u_i)n_i=-m \tag{4}$$

$u_i n_i$为边界的法向运动速度，i和j＝1,2,3。上面方程可以展开后线性化，例如压力P可展开为

$$P=P^{(0)}+P^{(1)}\varepsilon+P^{(2)}\varepsilon^2+\cdots\cdots \tag{5}$$

$$P^{(0)}=P^{(0,0)}+P^{(0,1)}M+\cdots\cdots \tag{6}$$

$$P^{(1)}=P^{(1,0)}+P^{(1,1)}M+\cdots\cdots \tag{7}$$

$P^{(0)}$表示没有扰动时的压力，$P^{(1)}$表示压力幅正比于小参数ε的线性扰动；$P^{(0,0)}$表示没有流动时的压力，$P^{(0,1)}$表示在有流动情况下与马赫数M成线性关系的压力；$P^{(1,0)}$表示没有流动时的压力扰动，$P^{(1,1)}$表示与马赫数成线性关系的压力扰动。其它参数可写为

$$T=T^{(0,0)}+T^{(0,1)}M+T^{(1,0)}\varepsilon+T^{(1,1)}\varepsilon M+\cdots\cdots \tag{8}$$

$$\rho=\rho^{(0,0)}+\rho^{(0,1)}M+\rho^{(1,0)}\varepsilon+\rho^{(1,1)}\varepsilon M+\cdots\cdots \tag{9}$$

$$v=v^{(0,1)}M+v^{(1,0)}\varepsilon+v^{(1,1)}\varepsilon M+\cdots\cdots \tag{10}$$

对于随时间变化的容腔，严格讲只有在流场中心$v^{(0,0)}=0$，在边界上$v^{(0,0)}$必然等于边界运动速度，即$\vec{v}^{(0,0)}\cdot\vec{n}=\vec{u}^{(0,0)}\cdot\vec{n}$。由于火药的燃速与流动速度$v^{(0,1)}$相比还是很小的，$v^{(0,0)}$在流场内的变化可近似地认为等于零。利用式(5)～(10)，通过数量级分析，方程(1)～(4)可保留其ε^1M^1阶次的各变量之间的关系为

$$\frac{\partial \rho^{(1,1)}}{\partial t}+\frac{\partial}{\partial x_j}(\rho^{(1,0)}v_j^{(0,1)}+\rho^{(0,0)}v_j^{(1,1)})=0 \tag{11}$$

$$\frac{\partial}{\partial t}(\rho^{(1,0)}v_i^{(0,1)}+\rho^{(0,0)}v_i^{(1,1)})+\frac{\partial}{\partial x_j}(\rho^{(0,0)}v_j^{(1,0)}v_i^{(0,1)}+\rho^{(0,0)}v_j^{(0,1)}v_i^{(1,0)})$$

$$+\frac{\partial P^{(1,1)}}{\partial x_i}=0 \tag{12}$$

$$\frac{\partial}{\partial t}(\rho^{(0,0)}e^{(1,1)}+\rho^{(1,1)}e^{(0,0)})+\frac{\partial}{\partial x_j}(\rho^{(1,0)}e^{(0,0)}v_j^{(0,1)}+\rho^{(0,0)}e^{(1,0)}v_j^{(0,1)}$$

$$+\rho^{(0,0)}e^{(0,0)}v_j^{(1,1)})+\frac{\partial}{\partial x_j}(P^{(1,0)}v_j^{(0,1)}+P^{(0,0)}v_j^{(1,1)})=0 \tag{13}$$

令 $$\rho u_i n_i=m_R \tag{14}$$

则有 $$m_R^{(1,1)}=\rho^{(0,0)}u_i^{(1,1)}n_i+\rho^{(1,0)}u_i^{(0,1)}n_i \tag{15}$$

$$-m^{(1,1)}=\rho^{(0,0)}v_i^{(1,1)}n_i+\rho^{(1,0)}v_i^{(0,1)}n_i-m_R^{(1,1)} \tag{16}$$

其它参数的扰动量在线性化后可分别写为

$$P^{(1,1)}=R(\rho^{(0,0)}T^{(1,1)}+\rho^{(1,1)}T^{(0,0)}) \tag{17}$$

$$P^{(1,0)}=R(\rho^{(1,0)}T^{(0,0)}+\rho^{(0,0)}T^{(0,1)}) \tag{18}$$

$$P^{(0,1)}=R(\rho^{(0,1)}T^{(0,0)}+\rho^{(0,0)}T^{(0,1)}) \tag{19}$$

$$e^{(1,0)}=C_vT^{(1,0)} \tag{20}$$

$$e^{(1,1)}=C_vT^{(1,1)}+v_j^{(0,1)}v_j^{(1,0)} \tag{21}$$

状态方程 $$P^{(0,0)}=R\rho^{(0,0)}T^{(0,0)} \tag{22}$$

利用式(17)～(22)，方程(13)可以简化为

$$\frac{\partial}{\partial t}(\rho^{(0,0)}v_j^{(0,1)}v_j^{(1,0)})+\frac{1}{\gamma-1}\frac{\partial P^{(1,1)}}{\partial t}+\frac{\gamma}{\gamma-1}\Big[\frac{\partial}{\partial x_j}(P^{(1,0)}v_j^{(0,1)}+$$

$$+P^{(0,0)}v_j^{(1,1)})\Big]=0 \tag{23}$$

把式(23)对时间微分并对式(12)求散度，再结合成波动方程为

$$\frac{\partial^2 P^{(1,1)}}{\partial x_i^2}-\frac{1}{(a^{(0,0)})^2}\frac{\partial^2 P^{(1,1)}}{\partial t^2}=\text{Ⓗ} \tag{24}$$

$$\text{Ⓗ}=-\frac{\partial}{\partial x_i}\frac{\partial}{\partial t}(\rho^{(1,0)}v_i^{(0,1)})-\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}[\rho^{(0,0)}(v_i^{(0,1)}v_j^{(1,0)}+v_i^{(1,0)}v_j^{(0,1)})]$$

$$+\frac{(\gamma-1)}{(a^{(0,0)})^2}\rho^{(0,0)}\frac{\partial^2}{\partial t^2}(v_i^{(0,1)}v_i^{(1,0)})+\frac{\gamma}{(a^{(0,0)})^2}\frac{\partial}{\partial t}\frac{\partial}{\partial x_j}(P^{(1,0)}v_j^{(0,1)}) \tag{25}$$

方程(2)～(4)的ε^1M^0阶次的变量关系式为

$$\frac{\partial}{\partial t}(\rho^{(0,0)}v^{(1,0)})+\frac{\partial P^{(1,0)}}{\partial x_i}=0 \tag{26}$$

$$\frac{\partial}{\partial t}(\rho^{(0,0)}e^{(1,0)}+\rho^{(1,0)}e^{(0,0)})+\frac{\partial}{\partial x_i}(\rho^{(0,0)}e^{(0,0)}v_j^{(1,0)})+$$

$$+\frac{\partial}{\partial x_j}(P^{(0.0)}v_j^{(1.0)})=0 \tag{27}$$

由于在边界上质量加入的速率正比于M，则有

$$\rho^{(0.0)}(v_i^{(1.0)}-u_i^{(1.0)})n_i=-m^{(1.0)}=0 \tag{28}$$

利用式(18)和(20)，式(26)和(27)可结合为一个齐次波动方程

$$\frac{\partial^2 P^{(1.0)}}{\partial x_i^2}=\frac{1}{(a^{(0.0)})^2}\frac{\partial^2 P^{(1.0)}}{\partial t^2} \tag{29}$$

$$v_i^{(1.0)}n_i=u_i^{(1.0)}n_i \tag{30}$$

由式（7）可知

$$\frac{\partial^2 P^{(1)}}{\partial x_i^2}-\frac{1}{(a^{(0.0)})^2}\frac{\partial^2 P^{(1)}}{\partial t^2}=M\text{㊐} \tag{31}$$

在控制边界上

$$\frac{\partial P^{(1)}}{\partial x_i}n_i=+\frac{\partial P^{(1.0)}}{\partial x_i}n_i-M\left\{\frac{\partial}{\partial t}(\rho^{(1.0)}v_i^{(0.1)}+\rho^{(0.0)}v_i^{(1.1)})n_i+\right.$$

$$\left.+\frac{\partial}{\partial x_j}[\rho^{(0.0)}(v_i^{(1.0)}v_i^{(0.1)}+v_j^{(0.1)}v_i^{(1.0)})]n_i\right\} \tag{32}$$

三、对于一维轴向截面随时间变化流管的简化

对于圆柱形装药的流管，设x_1为轴向坐标，x_2和x_3为垂直于轴向的坐标，通道某一截面A随轴向和时间变化。如果截面A的周长为S，则流动特性参数Q的平均形式为

$$\langle Q\rangle=\frac{1}{A}\int_A Q\,dA \tag{33}$$

$$\left\langle\frac{\partial Q}{\partial t}\right\rangle=\frac{1}{A}\frac{\partial}{\partial t}(A\langle Q\rangle) \tag{34}$$

$$\left\langle\frac{\partial Q}{\partial x_j}\right\rangle=\frac{1}{A}\int_A\frac{\partial Q}{\partial x_j}=\frac{1}{A}\oint Q\frac{n_j}{\sqrt{n_2^2+n_3^2}} \tag{35}$$

$$\left\langle\frac{\partial}{\partial x_j}(Qv_j)\right\rangle\qquad [j=1,2,3]$$

$$=\left\langle\frac{\partial}{\partial x_j}(Qv_j)\right\rangle+\left\langle\frac{\partial}{\partial x_1}(Qv_1)\right\rangle\quad [j=2,3]$$

$$=\frac{1}{A}\frac{\partial}{\partial x_1}(A\langle Qv_1\rangle)+\frac{1}{A}\oint Qv_j\frac{n_j}{\sqrt{n_2^2+n_3^2}}$$

$$[j=1,2,3] \tag{36}$$

利用式(33)～(36)，方程(31)可化为一维形式

$$\frac{\partial^2}{\partial x_1^2}(A\langle P^{(1)}\rangle)-\frac{2}{(a^{(0.0)})^2}\frac{\partial^2}{\partial t^2}(A\langle P^{(1)}\rangle)$$

$$=-\frac{\partial}{\partial x_1}\oint P^{(1)}\frac{n_1 ds}{\sqrt{n_2^2+n_3^2}}-\oint\frac{\partial P^{(1.0)}}{\partial x_j}\frac{n_j ds}{\sqrt{n_2^2+n_3^2}}+$$

$$+M\Big\{-\oint\frac{\partial P}{\partial x_i}^{(1.1)}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}-\frac{\partial^2}{\partial x\partial t}(A<\rho^{(1.0)}v^{(0.1)}>)-$$

$$-\frac{\partial^2}{\partial x_i^2}(2A<\rho^{(0.0)}v_1{}^{(0.1)}v_1{}^{(1.0)}>)+\frac{\gamma-1}{(a^{(0.0)})^2}\rho^{(0.0)}\times$$

$$\times\frac{\partial^2}{\partial t^2}(v<v_i^{(0.1)}v_1{}^{(1.0)}>)+\frac{\gamma}{(a^{(0.0)})^2}\frac{\partial^2}{\partial x_1\partial t}(A<P^{(1.0)}v_1^{(0.1)}>)$$

$$-\frac{\partial}{\partial t}\oint\rho^{(1.0)}v_i{}^{(0.1)}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}-\frac{\partial}{\partial x_1}\oint\rho^{(1.0)}(v_i^{(0.1)}v_1{}^{(1.0)}$$

$$+V_1{}^{(0.1)}v_i^{(1.0)})\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}-\frac{\partial}{\partial x}\oint\rho^{(0.0)}(v_i{}^{(0.1)}v_1{}^{(1.0)}+$$

$$+v_1{}^{(0.1)}v_i{}^{(1.0)})\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}-\oint\frac{\partial}{\partial x_i}\Big[\rho^{(0.0)}(v_i^{(0.1)}v_j^{(1.0)}+$$

$$+v_i^{(1.0)}v_j^{(0.1)})\Big]\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}+\frac{\gamma}{(a^{(0.0)})^2}\frac{\partial}{\partial t}\oint P^{(1.0)}v_i^{(0.1)}\times$$

$$\times\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}\Big\}\qquad(37)$$

利用式(18)，方程(37)右侧中的第八项和第十一项可以写为

$$-\frac{\partial}{\partial t}\oint\rho^{(1.0)}v_i^{(0.1)}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}+\frac{\gamma}{(a^{(0.0)})^2}\frac{\partial}{\partial t}\oint P^{(1.0)}v_i^{(0.1)}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}$$

$$=-\frac{\partial}{\partial t}\oint\frac{T^{(1.0)}}{T^{(0.0)}}(m^{(0.1)}-m_R^{(0.1)})\frac{ds}{\sqrt{n_2^2+n_3^2}}$$

$$=-\frac{\gamma-1}{(a^{(0.0)})^2}\frac{\partial}{\partial t}\oint[(m^{(0.1)}-m_R^{(0.1)})h^{(1.0)}]\frac{ds}{\sqrt{n_2^2+n_3^2}}\qquad(38)$$

利用式（12）和(16)，方程(37)右侧中的第三项和第十项可写为

$$-\oint\frac{\partial P^{(1.1)}}{\partial x_i}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}-\oint\frac{\partial}{\partial x_i}(\rho^{(0.0)}v_j^{(1.0)}v_i^{(0.1)}+\rho^{(0.0)}v_j^{(0.1)}v_i^{(1.0)})\times$$

$$\times\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}$$

$$=\oint\frac{\partial}{\partial t}(\rho^{(1.0)}v_i^{(0.1)}+\rho^{(0.0)}v_i^{(1.1)})\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}$$

$$=-\frac{(\gamma-1)}{(a^{(0.0)})^2}\oint\frac{\partial}{\partial t}[h^{(0.0)}(m^{(1.1)}-m_R^{(1.1)})]\frac{ds}{\sqrt{n_2^2+n_3^2}}\qquad(39)$$

方程(37)右侧第九项可变换为

$$-\frac{\partial}{\partial x_1}\oint\rho^{(0.0)}v\ (_i^{(0.1)}v_1{}^{(1.0)}+v_1{}^{(0.1)}v_i^{(1.0)})\frac{n_i ds}{\sqrt{n_2^2+n_3^2}}$$

$$=\frac{\partial}{\partial x_1}\oint m^{(0.1)}v_1{}^{(1.0)}\frac{ds}{\sqrt{n_2^2+n_3^2}}-\frac{\partial}{\partial x_1}\oint(m_R^{(0.1)}v_1{}^{(1.0)}+m_R^{(1.0)}v_1{}^{(0.1)}+$$

$$+m_R^{(0.0)}v_1{}^{(1.1)})\frac{ds}{\sqrt{n_2^2+n_3^2}}\qquad(40)$$

代入式(38)，(39)和(40)后，方程(37)变为

$$\frac{\partial^2}{\partial x_1^2}(A<P^{(1)}>) - \frac{1}{(a^{(0.0)})^2}\frac{\partial^2}{\partial t_2}(A<P^{(1)}>)$$

$$= -\frac{\partial}{\partial x_1}\oint P^{(1)}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}} - \oint \frac{\partial P}{\partial x_i}^{(1.0)}\frac{n_i ds}{\sqrt{n_2^2+n_3^2}} + M \times$$

$$\times\Big\{-\frac{\partial^2}{\partial x_1\partial t}(A<\rho^{(1.0)}v_1^{(0.1)}>) - \frac{\partial^2}{\partial x_1^2}(2A<\rho^{(0.0)}v_1^{(0.1)}v_1^{(1.0)}>) +$$

$$+\frac{\gamma}{(a^{(0.0)})^2}\frac{\partial^2}{\partial x_1\partial t}(A<P^{(1.0)}v_1^{(0.1)}>) + \frac{\gamma-1}{(a^{(0.0)})^2}\rho^{(0.0)}\frac{\partial^2}{\partial t^2}\times$$

$$\times(A<v_i^{(0.1)}v_i^{(1.0)}>) - \frac{\gamma-1}{(a^{(0.0)})^2}\oint\frac{\partial}{\partial t}[h^{(0.0)}(m^{(1.1)}-m_R^{(1.1)})]\times$$

$$\times\frac{ds}{\sqrt{n_2^2+n_3^2}} - \frac{\gamma-1}{(a^{(0.0)})^2}\frac{\partial}{\partial t}\oint[(m^{(0.1)}-m_R^{(0.1)})h^{(1.0)}]\frac{ds}{\sqrt{n_2^2+n_3^2}} -$$

$$-\frac{\partial}{\partial x_1}\oint m^{(0.1)}v_1^{(1.0)}\frac{ds}{\sqrt{n_2^2+n_3^2}} - \frac{\partial}{\partial x_1}\oint(m_R^{(0.1)}v_1^{(1.0)} + m_R^{(1.0)}v_1^{(0.1)} +$$

$$+m_R^{(0.0)}v_1^{(1.1)})\frac{ds}{\sqrt{n_2^2+n_3^2}}\Big\} \tag{41}$$

对于质量方程(1)，利用式(34)和(36)，再通乘以v_1则有

$$v_1\frac{\partial(A<\rho>)}{\partial t} + v_1\frac{\partial}{\partial x_1}(A<\rho v_1>) = v_1\oint\frac{m\,ds}{\sqrt{n_2^2+n_3^2}} - v_1\oint\frac{m_R ds}{\sqrt{n_2^2+n_3^2}} \tag{42}$$

上式线性化后成为:

$$v_1^{(0.1)}\frac{\partial(A<\rho^{(1.0)}>)}{\partial t} + v_1^{(1.1)}\frac{\partial(A<\rho^{(0.0)}>)}{\partial t} +$$

$$+v_1^{(1.0)}\frac{\partial}{\partial x_1}(A<\rho^{(0.0)}v_1^{(0.1)}>) + v^{(0.1)}\frac{\partial}{\partial x_1}(A<\rho^{(0.0)}v_1^{(1.0)}>)$$

$$= -v_1^{(0.1)}\oint\frac{m_R^{(1.0)}ds}{\sqrt{n_2^2+n_3^2}} + v_1^{(1.0)}\oint\frac{(m^{(0.1)}-m_R^{(0.1)})ds}{\sqrt{n_2^2+n_3^2}} -$$

$$-v_1^{(1.1)}\oint\frac{m_R^{(0.0)}ds}{\sqrt{n_2^2+n_3^2}} \tag{43}$$

利用式(43)则有

$$-\frac{\partial^2}{\partial x\partial t}(A<\rho^{(1.0)}v_1^{(0.1)}>) - \frac{\partial^2}{\partial x_1^2}(2A<\rho^{(0.0)}v_1^{(0.1)}v_1^{(1.0)}>)$$

$$= -\frac{\partial}{\partial x_1}\left[v_1^{(1.1)}\frac{\partial}{\partial t}(A<\rho^{(0.0)}>)\right] - \rho^{(0.0)}\frac{\partial A}{\partial x_1}\frac{\partial<v_1^{(0.1)}v_1^{(1.0)}}{\partial x_1} -$$

$$-A\rho^{(0.0)}\frac{\partial^2}{\partial x_1^2}<v_1^{(0.1)}v_1^{(1.0)}> - \frac{\partial}{\partial x_1}\left[v_1^{(1.0)}\oint\frac{(m^{(0.1)}-m_R^{(0.1)})ds}{\sqrt{n_2^2+n_3^2}}\right] +$$

$$+\frac{\partial}{\partial x_1}\left(v_1^{(0.1)}\oint\frac{m_R^{(1.1)}ds}{\sqrt{n_2^2+n_3^2}} + v_1^{(1.1)}\oint m_R^{(0.0)}\frac{ds}{\sqrt{n_2^3+n_3^2}}\right) \tag{44}$$

注意到

$$\frac{\partial(A<\rho^{(0.0)}>)}{\partial t}=-\oint m_R^{(0.0)}\frac{ds}{\sqrt{n_2^2+n_3^2}};\quad \frac{\partial A}{\partial t}=\oint R ds$$

R 为装药燃速，即为边界运动速度u_i在与轴向垂直方向的 投影。如果 沒有 侵蚀 燃烧，$\frac{\partial R}{\partial x_1}=0$；如果稳态流速沿轴向分布为常数，$\frac{\partial v_1}{\partial x_1}^{(0.1)}=0$。方程（41）中右侧第五项可写为

$$\frac{\gamma}{(a^{(0.0)})^2}\frac{\partial^2}{\partial x_1\partial t}(A<P^{(1.0)}v_1^{(0.1)}>)$$

$$=\frac{\gamma}{(a^{(0.0)})^2}\Big[A<v_1^{(0.1)}>\frac{\partial}{\partial x_1}\frac{\partial<P^{(1.0)}}{\partial t}+\frac{\partial<P^{(1.0)}>}{\partial t}\frac{\partial}{\partial x_1}(A<v_1^{(0.1)}>)+$$

$$+\frac{\partial<P^{(1.0)}>}{\partial x_1}v_1^{(0.1)}\oint\frac{Rds}{\sqrt{n_2^2+n_3^2}}\Big]\tag{45}$$

在一维流场中，方程(41)右侧的第六项可写为

$$\frac{\gamma-1}{(a^{(0.0)})^2}\rho^{(0.0)}\frac{\partial^2}{\partial t^2}(A<v_1^{(0.1)}v_1^{(1.0)}>)$$

$$=\frac{\gamma-1}{(a^{(0.0)})^2}\rho^{(0.0)}(2<v_1^{(0.1)}>\frac{\partial<v_1^{(1.0)}>}{\partial t}\oint\frac{Rds}{\sqrt{n_2^2+n_3^2}}+A<v_1^{(0.1)}>\times$$

$$\times\frac{\partial^2<v_1^{(1.0)}>}{\partial t^2}\Big)\tag{46}$$

方程（41）左侧的两项可分别写为

$$\frac{\partial^2}{\partial t^2}\ (A<P^{(1)}>)\ =2\frac{\partial<P^{(1)}>}{\partial t}\oint\frac{Rds}{\sqrt{n^2{}_2+n^3{}_3}}+A\frac{\partial^2<P^{(1)}>}{\partial t^2}\tag{47}$$

$$\frac{\partial^2}{\partial x^2{}_1}\ (A<P^{(1)}>)\ =\frac{\partial}{\partial x_1}\Big(A\frac{\partial<P^{(1)}>)}{\partial x_1}\Big)+\frac{\partial<P^{(1)}>}{\partial x_1}\ \frac{\partial A}{\partial x_1}\tag{48}$$

由于针对一维平均流，下面略去平均符号。注意到压力复振 幅 $P^{(1)}(x_1,\ t)=\hat{P}^{(1)}(x_1)\exp(iKa^{(0.0)}t)$，其它参数也同样展开。把式（44）～（48）都代入方程（41）中，则有

$$\frac{1}{A}\frac{\partial}{\partial x_1}\Big(A\frac{\partial\hat{P}^{(1)}}{\partial x_1}\Big)+K^2\hat{P}^{(1)}=I_0+\sum_{i=}^{4}MI_i\tag{49}$$

$$I_0=-\frac{1}{A}\ \frac{\partial}{\partial x_1}\oint\hat{P}^{(1)}\frac{n_1ds}{\sqrt{n^2{}_2+n^2{}_3}}-\frac{1}{A}\oint\frac{\partial\hat{P}_{(1.0)}}{\partial x_1}\frac{n_1ds}{\sqrt{n^2{}_2+n^2{}_3}}-$$

$$-\frac{1}{A}\ \frac{\partial\hat{P}^{(1)}}{\partial x_1}\ \frac{dA}{dx_1}$$

$$I_1=-\rho^{(0.0)}\frac{\partial(v_1^{(0.1)}\hat{v}_1^{(1.0)})}{\partial x_1}\frac{d\ln A}{dx_1}-\rho^{(0.0)}\frac{\partial^2}{\partial x^2{}_1}(v_1^{(0.1)}\times$$

$$\times \hat{v}_1{}^{(1.0)}) - (\gamma - 1) K_e{}^2 \rho^{(0.0)} v_1{}^{(0.1)} \hat{v}_1{}^{(1.0)} + \frac{i\gamma K_e}{(a^{(0.0)})} v_1{}^{(0.1)} \times$$

$$\times \frac{\partial \hat{P}^{(1.0)}}{\partial x_1} + \frac{i\gamma K_e}{a^{(0.0)} A} \hat{P}^{(1.0)} \frac{\partial}{\partial x_1} (A v_1{}^{(0.1)})$$

$$I_2 = -\frac{1}{A} \frac{\partial}{\partial x_1} \left(\rho^{(0.0)} \ \hat{v}_1{}^{(1.1)} \oint \frac{R ds}{\sqrt{n^2{}_2 + n^2{}_3}} \right) + \frac{\gamma}{(a^{(0.0)})^2} \times$$

$$\times \frac{v_1{}^{(0.1)}}{A} \frac{\partial \hat{P}^{(1.0)}}{\partial x_1} \oint \frac{R ds}{\sqrt{n^2{}_2 + n^2{}_3}} + \frac{2iKe(\gamma - 1)}{a^{(0.0)} A} \rho^{(0.0)} v_1{}^{(0.1)} \times$$

$$\times \hat{v}_1{}^{(1.0)} \oint \frac{R ds}{\sqrt{n^2{}_2 + n^2{}_3}} + \frac{2iKe}{a^{(0.0)} A} \hat{P}^{(1)} \oint \frac{R ds}{\sqrt{n^2{}_2 + n^2{}_3}} +$$

$$I_3 = -\frac{iKe(\gamma - 1)}{a^{(0.0)} A} \left[\oint h^{(0.0)} \left(\hat{m}^{(1.1)} - \hat{m}_R^{(1.1)} \right) \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} + \right.$$

$$\left. + \oint h^{(1.0)} \left(m^{(0.1)} - m_R^{(0.1)} \right) \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} \right]$$

$$I_4 = \frac{1}{A} \frac{\partial}{\partial x_1} \left[\oint \hat{v}_1{}^{(1.0)} \left(m^{(0.1)} - m_R^{(0.1)} \right) \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} - \hat{v}_1{}^{(1.0)} \times \right.$$

$$\left. \times \oint \left(m^{(0.1)} - m_R^{(0.1)} \right) \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} \right] - \frac{1}{A} \frac{\partial}{\partial x_1} \left[\left(\oint v_1{}^{(0.1)} \hat{m}_R^{(1.0)} - \right. \right.$$

$$\left. \left. - v_1{}^{(0.1)} \oint \hat{m}_R^{(1.0)} \right) \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} \right] -$$

$$- \frac{1}{A} \frac{\partial}{\partial x_1} \left[\left(\oint \hat{v}_1{}^{(1.1)} m_R^{(0.0)} - \hat{v}_1{}^{(1.1)} \oint m_R^{(0.0)} \right) \times \right.$$

$$\left. \times \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} \right]$$

对于方程（12）取平均过程可写为：

$$M \left[\frac{1}{A} \frac{\partial}{\partial t} \left(A \langle \rho^{(1.0)} v_1{}^{(0.1)} + \rho^{(0.0)} v_1{}^{(1.1)} \rangle \right) + \frac{1}{A} \frac{\partial}{\partial x_1} \times \right.$$

$$\times \left(2A \ \langle \rho^{(0.0)} v_1{}^{(0.1)} v_1{}^{(1.0)} \rangle \right) + \frac{1}{A} \oint \rho^{(0.0)} \left(v_i{}^{(0.1)} v_1{}^{(1.0)} + \right.$$

$$\left. \left. + v_i{}^{(1.0)} v_1{}^{(0.1)} \right) \frac{n_i ds}{\sqrt{n^2{}_2 + n^2{}_3}} \right] + \frac{1}{A} \frac{\partial (A \ \langle P^{(1)} \rangle)}{\partial x_1} + \frac{1}{A} \times$$

$$\times \oint P^{(1)} \frac{n_i ds}{\sqrt{n^2{}_2 + n^2{}_3}} = 0 \qquad (50)$$

把式（50）展开后并利用式（4）和（14），则有

$$M \left\{ v_1{}^{(0.1)} \frac{\partial (A \langle \rho^{(1.0)} \rangle)}{\partial t} + v_1{}^{(1.1)} \frac{\partial (A \langle \rho^{(0.0)} \rangle)}{\partial t} + A \rho^{(0.0)} \times \right.$$

$$\times \frac{\partial \langle v_1^{(1.1)}\rangle}{\partial t} + v_1^{(1.0)}\frac{\partial}{\partial x_1}\left(A \ \langle \rho^{(0.0)} v_1^{(0.1)}\rangle \right) + v_1^{(0.1)}\times$$

$$\times \frac{\partial}{\partial x_1}\left(A \ \langle \rho^{(0.0)} v_1^{(1.0)}\rangle \right) + A\rho^{(0.0)}\frac{\partial}{\partial x_1}\langle v_1^{(0.1)} v_1^{(1.0)}\rangle$$

$$+ \oint v_1^{(0.1)} m_R^{(1.0)} \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} + \oint v_1^{(1.1)} m_R^{(0.0)} \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} -$$

$$- \oint v_1^{(1.0)}\left(m^{(0.1)} - m_R^{(0.1)}\right)\frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}}\Bigg\} + \frac{\partial}{\partial x_1}\left(A \ \langle P^{(1)}\rangle \right) +$$

$$+ \oint \frac{P^{(1)} n_1 ds}{\sqrt{n^2{}_2 + n^2{}_3}} = 0 \tag{51}$$

利用式（43），上式可以展开成为圆柱形燃烧室压力扰动纵向振荡在两端的边界条件：

$$\left.\frac{\partial \hat{P}^{(1)}}{\partial x_1}\right|_{x_1 = 0}^{x_1 = L} = -I_{A0} - M[I_{A1} + I_{A4} + I_{A5}] \tag{52}$$

$$I_{A0} = \frac{1}{A}\oint \frac{\hat{P}^{(1)} n_1 ds}{\sqrt{n^2{}_2 + n^2{}_3}} + \hat{P}^{(1)} \ \frac{d l_n A}{d x_1}$$

$$I_{A1} = \rho^{(0.0)}\frac{\partial}{\partial x_1}\left(v_1^{(0.1)} \hat{v}_1^{(1.0)}\right)$$

$$I_{A4} = \frac{1}{A} \ \hat{v}_1^{(1.0)} \oint\left(m^{(0.1)} - m_R^{(0.1)}\right)\frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} - \frac{1}{A}\times$$

$$\times \oint \hat{v}_1^{(1.0)}\left(m^{(0.1)} - m_R^{(0.1)}\right)\frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} - \frac{1}{A}\Bigg(v_1^{(0.1)}\times$$

$$\times \oint \hat{m}_R^{(1.0)} \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} - \oint v_1^{(0.1)} \hat{m}_R^{(1.0)} \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}}\Bigg) -$$

$$- \frac{1}{A}\Bigg(\hat{v}_1^{(1.1)} \oint m_R^{(0.0)} \ \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}} - \oint \hat{v}_1^{(1.1)} m_R^{(0.0)} \times$$

$$\times \frac{ds}{\sqrt{n^2{}_2 + n^2{}_3}}\Bigg)$$

$$I_{A5} = \rho^{(0.0)} \ \frac{\partial v_1^{(1.1)}}{\partial t} = i\rho^{(0.0)} a^{(0.0)} Ke \ \hat{v}_1^{(1.1)}$$

设有扰动时的经典亥姆霍兹方程为

$$\begin{cases} \dfrac{1}{A}\dfrac{d}{d x_1}\left(A \dfrac{d \hat{P}_e^{(1.0)}}{d x_1}\right) + K_e^2 \hat{P}_e^{(1.0)} = 0 \\ \dfrac{d \hat{P}_e^{(1.0)}}{d x_1} = 0 \qquad (x_1 = 0, \ L) \end{cases} \tag{53}$$

由 $K = K^{(0.0)} + MK^{(0.1)}$，则可得出复波数解：

$$(K^{(0\cdot1)})^2=(K_e^{(0\cdot1)})^2+\frac{1}{E_e^2}\left\{\int_0^{L_4}\sum_{i=0}I_i\hat{P}_e^{(1\cdot0)}Adx_1+\hat{P}_e^{(1\cdot0)}A\times\right.$$
$$\left.\times[I_{A0}+I_{A1}+I_{A4}+I_{A5}]_0^L\right\} \tag{54}$$

其中$E_e^2=\int_0^L(\hat{P}_e^{(1\cdot0)})^2dx_1$

对于管形装药，在未发生侵蚀燃烧情况下可以认为$\sqrt{n^2{}_2+n^2{}_3}=1$。方程（54）解得的结果为

$$[(K^{(0\cdot1)})^2-(K_e^{(0\cdot1)})^2]E_e^2$$
$$=i\rho^{(0\cdot0)}a^{(0\cdot0)}K_e^{(0\cdot0)}\left\{\left[\hat{P}_e^{(1\cdot0)}\left(\hat{v}_1{}^{(1\cdot1)}+\frac{v_1{}^{(0\cdot1)}\hat{P}_e^{(1\cdot0)}}{\rho^{(0\cdot0)}(a^{(0\cdot0)})^2}\right)\times\right.\right.$$
$$\left.\times A\right]^{(\mathrm{I})}-\int_0^L\hat{P}_e^{(1\cdot0)}\oint\left[\frac{\hat{m}^{(1\cdot1)}-\hat{m}_R^{(1\cdot1)}}{\rho^{(0\cdot0)}}+\frac{(m^{(0\cdot1)}-m_R^{(0\cdot1)})}{\rho^{(0\cdot0)}}\times\right.$$
$$\left.\left.\times\frac{\Delta\hat{T}^{(1\cdot0)}}{T^{(0\cdot0)}}\right]^{(\mathrm{II})}dsdx_1\right\}-\left\{\int_0^L\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\oint\hat{v}_1{}^{(1\cdot0)}\left(m^{(0\cdot1)}-m_R^{(0\cdot1)}\right)\times\right.$$
$$\times dsdx_1-\int_0^L\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\left(\oint\hat{v}_1{}^{(1\cdot1)}m_R^{(0\cdot0)}-\hat{v}_1{}^{(1\cdot1)}\int m_R^{(0\cdot0)}\right)\times$$
$$\times dsdx_1-\int_0^L\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\hat{v}_1{}^{(1\cdot0)}\oint\left(m^{(0\cdot1)}-m_R^{(0\cdot1)}\right)dsdx_1-$$
$$-\int_0^L\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\left(\oint v_1{}^{(0\cdot1)}\hat{m}_R^{(1\cdot0)}-v_1{}^{(0\cdot1)}\oint\hat{m}_R{}^{(1\cdot0)}\right)\times$$
$$\left.\times dsdx_1\right\}^{(\mathrm{III})}+\left\{\int_0^L\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\rho^{(0\cdot0)}\hat{v}_1{}^{(1\cdot1)}\oint Rdsdx_1+\right.$$
$$+\frac{3\gamma-1}{(a^{(0\cdot0)})^2}\int_0^L v_1{}^{(0\cdot1)}\hat{P}_e^{(1\cdot0)}\frac{d\hat{P}^{(1\cdot0)}}{dx_1}\oint Rdsdx_1+\frac{2iK_e{}^{(0\cdot0)}}{a^{(0\cdot0)}}\times$$
$$\left.\times\int_0^L(\hat{P}_e^{(1\cdot0)})^2\oint Rdsdx_1\right\}^{(\mathrm{IV})}+\left\{\int_0^L\hat{P}_e^{(1\cdot0)}\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\times\right.$$
$$\left.\times\frac{dA}{dx_1}dx_1\right\}^{(\mathrm{V})} \tag{55}$$

复波数解各项的主要物理含义是

（Ⅰ）表示管状药两端截面导纳函数和平均流的作用；

（Ⅱ）侧燃烧表面响应函数和非等熵温度扰动的作用；

（Ⅲ）质量加入与平均流轴向边界动量相互作用的影响；

（Ⅳ）燃烧边界运动和空间扩大的影响；

（Ⅴ）侧表面压力的轴向分量对声场所作功的影响。Culick忽略了这一项的作用，本文出现该项是由于对三维方程取平均过程产生的自然结果，当然也可仿照Culick略去不计。

四、结果与讨论

如果假设通道截面不随时间改变，装药燃速R = 0，略去侧表面轴向压力分量，则式(55)可以进一步简化为

$$\left[(K^{(0\cdot1)})^2-(K_e^{(0\cdot1)})_e^2\right]E_e^2$$

$$=i\rho^{(0\cdot0)}a^{(0\cdot0)}K_e{}^{(0\cdot0)}\left\{\left[\hat{P}_e^{(1\cdot0)}\left(\hat{v}_1^{(1\cdot1)}+\frac{v_1^{(0\cdot1)}\hat{P}_e^{(1\cdot0)}}{\rho^{(0\cdot0)}(a^{(0\cdot0)})^2}\right)A\right]_O^L-\right.$$

$$\left.-\int_O^L\hat{P}_e^{(1\cdot0)}\oint\left[\frac{\hat{m}^{(1\cdot1)}}{\rho^{(0\cdot0)}}+\frac{m^{(0\cdot1)}}{\rho^{(0\cdot0)}}\frac{\Delta\hat{T}^{(1\cdot0)}}{T^{(0\cdot0)}}\right]dsdx_1\right\}-$$

$$-\int_O^L\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\oint\hat{v}_1^{(1\cdot0)}m^{(0\cdot1)}dsdx_1+\frac{i}{\rho^{(0\cdot0)}a^{(0\cdot0)}K_e{}^{(0\cdot0)}}\times$$

$$\times\int_O^L\left(\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\right)^2\oint m^{(0\cdot1)}dsdx_1 \tag{56}$$

上式与Culick在文献〔1〕中导出的式(44)完全一致(不考虑颗粒物质)。由

$$-\alpha=\frac{a^{(0\cdot0)}}{2iK_e^{(0\cdot0)}E_e^2}\left[(K^{(0\cdot1)})^2-(K_e^{(0\cdot1)})^2\right]$$

可知，在式(55)中由于燃烧边界运动产生的项 $\frac{3\gamma-1}{(a^{(0\cdot0)})^2}\int_O^L v_1{}^{(0\cdot1)}\hat{P}_e{}^{(1\cdot0)}\frac{dP_e{}^{(1\cdot0)}}{dx_1}\times$

$\times\oint Rdsdx_1$对声能增长时的频率产生影响，而对声能增长率不起作用。设

$$\int_O^L P^{(0\cdot0)}\hat{v}_1{}^{(1\cdot1)}\frac{d\hat{P}_e{}^{(1\cdot0)}}{dx_1}\oint Rdsdx_1+\frac{2iK_e}{a^{(0\cdot0)}}\int_O^L(\hat{P}_e{}^{(1\cdot0)})^2\oint Rdsdx_1$$

对声能增长率的作用为α_R，则有

$$-\alpha_R=\frac{1}{E_e^2}\int_O^L\left[\frac{1}{2K_e^2}\left(\frac{d\hat{P}_e{}^{(1\cdot0)}}{dx_1}\right)^2+(\hat{P}_e{}^{(1\cdot0)})^2\right]\oint Rdsdx_1 \tag{57}$$

上式表示了声场中单位体积内的动能$\frac{1}{2K_e^2}\left(\frac{d\hat{P}_e^{(1\cdot0)}}{dx_1}\right)^2$和势能$(\hat{P}_e{}^{(1\cdot0)})^2$在燃烧中扩大的体积内$\int_O^L\oint Rdsdx_1$的积分之和。它代表这部分体积中声场声能的时间平均密度。当考虑装药燃烧造成容腔变化时，整个燃烧室单位体积内的声能将发生改变。可以得出结论：装药烧去部分的空间瞬态扩大和边界的运动平衡掉了总声能的一部分。α_R对于总声能变化定量的影响尚需进一步研究和实验。从现在推出的公式可以看出，压力愈高，燃速愈快，对声能增长起到的抵销作用也愈大。

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